ASSIGNMENT: CLASS XI, CH: TRIGONOMETRIC FUNCTIONS

BASED ON RADIAN MEASURE OF AN ANGLE

1. *Find the length of an arc of a circle of radius 5 cm subtending a central angle measuring* 15°. *(Ans:* $5\pi/12$*)*
2. *Find in degrees the angle subtended at the centre of a circle of diameter 50cm by an arc of length 11 cm. (Ans:* $25^\circ 12'$*)*
3. *A horse is tied to a post by a rope. If the horse moves along a circular path always keeping the rope tight and describes 88 meters when it has traced out* 72° *at the centre, find the length of the rope. (Ans: 70 meters)*
4. *A circular wire of radius 7.5cm is cut and bent so as to lie along the circumference of a hoop whose radius is 125 cm. Find in degrees the angle which is subtended at the centre of the hoop. (Ans:* $22^\circ 30'$*)*
5. *The moon's distance from the earth is 360000kms and its diameter subtends an angle of* $31'$ *at the eye of the observer. Find the diameter of the moon. (Ans: 3247.62km)*
6. *If the angular diameter of the moon be* $30'$*, how far from the eye a coin of diameter 2.2cm be kept to hide the moon?(Ans: 252cm)*
7. *Assuming that a person of normal sight can read at such a distance that the letters subtended an angle of* $5'$ *at his eye, find what is the height of the letters that he can read at a distance of 12 meters.(Ans: 1.7cm)*
8. *Find the angle between the minute hand of a clock and the hour hand when the time is 7:20 AM (Ans:* 100°*)*
9. *Find in degrees and radians the angle between the hour hand and minute hand of a clock at half past three.(Ans:* $5\pi/12$*)*
10. *A railway train is travelling on a circular curve of 1500 meters radius at the rate of 66 km/hr. Through what angle has it turned in 10 seconds? (Ans:* $\left(\frac{11}{90}\right)^c$*)*
11. *Find the diameter of the sun in km supposing that it subtends an angle of* $32'$ *at the eye of an observer. Given that the distance of the sun is* 91×10^6 *km.(Ans: 847407.4 km)*

BASED ON ALLIED ANGLES
&
EVALUATION OF VALUES AT VARIOUS ANGLES

12. *Find the value of the following:*
    (i) $\sin 315^\circ$ *(ii)* $\cos 210^\circ$ *(iii)* $\cos\left(-480^\circ\right)$ *(iv)* $\sin\left(-1125^\circ\right)$ *(v)* $\operatorname{cosec} 390^\circ$ *(vi)* $\cot 570^\circ$
    (vii) $\operatorname{cosec}\left(-1200^\circ\right)$ *(viii)* $\cos 855^\circ$ *(ix)* $\sin 1845^\circ$ *(x)* $\cos 1755^\circ$ *(xi)* $\sin 4530^\circ$
    (Ans: $-1/\sqrt{2}, -\sqrt{3}/2, -1/2, -1/\sqrt{2}, 2, \sqrt{3}, -2/\sqrt{3}, -1/\sqrt{2}, 1/\sqrt{2}, 1/\sqrt{2}, -1/2$*)*

13. *Prove that* $\tan\frac{11\pi}{3}-2\sin\frac{4\pi}{6}-\frac{3}{4}\cos ec^2\frac{\pi}{4}+4\cos^2\frac{17\pi}{6}=\frac{3-4\sqrt{3}}{2}$.

14. *Prove that* $\frac{\cos ec\left(90^\circ+\theta\right)+\cot\left(450^\circ+\theta\right)}{\cos ec\left(90^\circ-\theta\right)+\tan\left(180^\circ-\theta\right)}+\frac{\tan\left(180^\circ+\theta\right)+\sec\left(180^\circ-\theta\right)}{\tan\left(360^\circ+\theta\right)-\sec\left(-\theta\right)}=2$

15. *Prove that* $\left\{1+\cot\theta-\sec\left(\frac{\pi}{2}+\theta\right)\right\}\left\{1+\cot\theta+sec\left(\frac{\pi}{2}+\theta\right)\right\}=2\cot\theta$.

16. *Prove that* $\frac{\cos\left(2\pi+\theta\right)\cos ec\left(2\pi+\theta\right)\tan\left(\pi/2+\theta\right)}{\sec\left(\pi/2+\theta\right)\cos\theta\cot\left(\pi+\theta\right)}=1$

BASED ON ANGLE'S SUM FORMULAE

17. *If* $\cos\left(\alpha+\beta\right)=\frac{4}{5},\sin\left(\alpha-\beta\right)=\frac{5}{13}$ *and* α , β *lie between 0 and* $\frac{\pi}{4}$, *prove that* $\tan 2\alpha=\frac{56}{33}$.

18. *Prove that* $\tan 70^\circ=\tan 20^\circ+2\tan 50^\circ$.

19. *If* $\tan\left(\alpha+\theta\right)=n\tan\left(\alpha-\theta\right)$, *show that* $\left(n+1\right)\sin 2\theta=\left(n-1\right)\sin 2\alpha$.

20. *If* $\sin\alpha+\sin\beta=a$ *and* $\cos\alpha+\cos\beta=b$, *show that*

(i) $\cos\left(\alpha+\beta\right)=\frac{b^2-a^2}{b^2+a^2}$ *(ii)* $\sin\left(\alpha-\beta\right)=\frac{2ab}{a^2+b^2}$

21. *If* α *and* β *are the solutions of the equation* $a\tan\theta+b\sec\theta=c$ *,then show that* $\tan\left(\alpha+\beta\right)=\frac{2ac}{a^2-c^2}$.

22. *Prove that* $\frac{\cos 9^\circ+\sin 9^\circ}{\cos 9^\circ-\sin 9^\circ}=\tan 56^\circ$.

23. *If* $\tan A=\frac{5}{6}$ *and* $\tan B=\frac{1}{11}$, *prove that* $A+B=\frac{\pi}{4}$.

24. *Prove the following*

(i) $\frac{\sin\left(A-B\right)}{\sin A\sin B}+\frac{\sin\left(B-C\right)}{\sin B\sin C}+\frac{\sin\left(C-A\right)}{\sin C\sin A}=0$

(ii) $\tan 8\theta-\tan 6\theta-\tan 2\theta=\tan 8\theta\tan 6\theta\tan 2\theta$

(iii) $\tan 15^\circ+\tan 30^\circ+\tan 15^\circ\tan 30^\circ=1$

(iv) $\frac{\tan^2 2\theta-\tan^2\theta}{1-\tan^2 2\theta\tan^2\theta}=\tan 3\theta\tan\theta$

25. *If* $\tan A=x\tan B$, *prove that* $\frac{\sin\left(A-B\right)}{\sin\left(A+B\right)}=\frac{x-1}{x+1}$.

26. *If* $\tan x+\tan\left(x+\frac{\pi}{3}\right)+\tan\left(x+\frac{2\pi}{3}\right)=3$, *then prove that* $\frac{3\tan x-\tan^3 x}{1-3\tan^2 x}=1$

BASED ON PRODUCT FORMULAE

27. Prove the following:

(i) $\cos 20^\circ \cos 40^\circ \cos 60^\circ \cos 80^\circ = \frac{1}{16}$

(ii) $\sin 10^\circ \sin 30^\circ \sin 50^\circ \sin 70^\circ = \frac{1}{16}$

(iii) $\sin 20^\circ \sin 40^\circ \sin 60^\circ \sin 80^\circ = \frac{3}{16}$

(iv) $\tan 20^\circ \tan 40^\circ \tan 80^\circ = \tan 60^\circ = \sqrt{3}$

(v) $\cos 10^\circ \cos 30^\circ \cos 50^\circ \cos 70^\circ = \frac{3}{16}$

28. Without calculating the values of cos75^{O}and cos15^O, find the value of $\cos 75^\circ \cos 15^\circ$.

29. Prove that $2\sin\frac{5\pi}{12}\cos\frac{\pi}{12} = \frac{\sqrt{3}+2}{2}$.

BASED ON SUM FORMULAE

30. Prove the following:

(i) $\sin\alpha + \sin\left(\alpha + 2\pi/3\right) + \sin\left(\alpha + 4\pi/3\right) = 0$

(ii) $\cos\alpha + \cos\beta + \cos\gamma + \cos\left(\alpha+\beta+\gamma\right) = 4\cos\frac{\alpha+\beta}{2}\cos\frac{\beta+\gamma}{2}\cos\frac{\gamma+\alpha}{2}$

(iii) $\frac{\cos 2A\cos 3A - \cos 2A\cos 7A + \cos A\cos 10A}{\sin 4A\sin 3A - \sin 2A\sin 5A + \sin 4A\sin 7A} = \cot 6A\cot 5A$

(iv) $\frac{\sin\left(A-C\right) + 2\sin A + \sin\left(A+C\right)}{\sin\left(B-C\right) + 2\sin B + \sin\left(B+C\right)} = \frac{\sin A}{\sin B}$

(v) $\sin A + \sin 2A + \sin 4A + \sin 5A = 4\cos\frac{A}{2}\cos\frac{3A}{2}\sin 3A$

(vi) $\sin\frac{\theta}{2}\sin\frac{7\theta}{2} + \sin\frac{3\theta}{2}\sin\frac{11\theta}{2} = \sin 2\theta\sin 5\theta$

31. If $\sin 2A = \lambda\sin 2B$ *, prove that:* $\frac{\tan\left(A+B\right)}{\tan\left(A-B\right)} = \frac{\lambda+1}{\lambda-1}$

32. If $\frac{\sin\left(\theta+\alpha\right)}{\cos\left(\theta-\alpha\right)} = \frac{1-m}{1+m}$ *, prove that:* $\tan\left(\frac{\pi}{4}-\theta\right)\tan\left(\frac{\pi}{4}-\alpha\right) = m$.

BASED ON DOUBLE, TRIPPLE, HALF ANGLE FORMULA

33. Prove the following:

(i) $\frac{\sin 2\theta}{1-\cos 2\theta} = \cot\theta$

(ii) $\dfrac{1+\sin\theta-\cos\theta}{1+\sin\theta+\cos\theta}=\tan\dfrac{\theta}{2}$

(iii) $\dfrac{\cos\theta}{1+\sin\theta}=\tan\left(\dfrac{\pi}{4}-\dfrac{\theta}{2}\right)$

(iv) $\sqrt{2+\sqrt{2+\sqrt{2+2\cos 8\theta}}}=2\cos\theta$

(v) $\dfrac{\sec 8\theta-1}{\sec 4\theta-1}=\dfrac{\tan 8\theta}{\tan 2\theta}$

(vi) $\left(1+\cos\dfrac{\pi}{8}\right)\left(1+\cos\dfrac{3\pi}{8}\right)\left(1+\cos\dfrac{5\pi}{8}\right)\left(1+\cos\dfrac{7\pi}{8}\right)=\dfrac{1}{8}$

(vii) $\cos 5A=16\cos^5 A-20\cos^3 A+5\cos A$

(viii) $\cos^3 A+\cos^3\left(120^\circ+A\right)+\cos^3\left(240^\circ+A\right)=\dfrac{3}{4}\cos 3A$

34. Find the value of $\cos\left(22\dfrac{1}{2}\right)^\circ,\sin\left(22\dfrac{1}{2}\right)^\circ,\tan\left(22\dfrac{1}{2}\right)^\circ,\sin\left(7\dfrac{1}{2}\right)^\circ,\cos\left(7\dfrac{1}{2}\right)^\circ$

(Ans: $\sqrt{\dfrac{\sqrt{2}+1}{2\sqrt{2}}},\sqrt{\dfrac{\sqrt{2}-1}{2\sqrt{2}}},\sqrt{2}-1,\dfrac{\sqrt{4-\sqrt{6}-\sqrt{2}}}{2\sqrt{2}},\dfrac{\sqrt{4+\sqrt{6}+\sqrt{2}}}{2\sqrt{2}}$ *)*

BASED ON GENERAL SOLUTION OF TRIGONOMETRICAL EQUATIONS

35. Solve the following trigonometric equations:

(i) $\sin\theta+\sin 3\theta+\sin 5\theta=0$ *(Ans:* $\theta=\dfrac{n\pi}{3}$ *or* $\theta=m\pi\pm\dfrac{\pi}{3}$ *, where,* $m,n\in Z$ *)*

(ii) $\sin m\theta+\sin n\theta=0$ *(Ans:* $\theta=\dfrac{2r\pi}{m+n}$ *or* $\theta=\dfrac{(2s+1)\pi}{m-n}$ *, where,* $r,s\in Z$ *)*

(iii) $2\tan\theta-\cot\theta=-1$ *(Ans:* $\theta=n\pi-\dfrac{\pi}{4}$ *or* $\theta=m\pi+\alpha$ *, where* $m,n\in Z$ *and* $\tan\alpha=\dfrac{1}{2}$ *)*

(iv) $\cot^2\theta+\dfrac{3}{\sin\theta}+3=0$ *(Ans:* $\theta=n\pi+(-1)^{n+1}\dfrac{\pi}{6}$ *or,* $\theta=m\pi+(-1)^{m+1}\dfrac{\pi}{2}$ *,* $m,n\in Z$ *)*

(v) $\tan\theta+\tan 2\theta+\tan 3\theta=\tan\theta\tan 2\theta\tan 3\theta$ *(Ans:* $\theta=\dfrac{n\pi}{3}, n\in Z$ *)*

(vi) $\tan\theta+\tan 2\theta+\sqrt{3}\tan\theta\tan 2\theta=\sqrt{3}$ *(Ans:* $\theta=\dfrac{n\pi}{3}+\dfrac{\pi}{9}, n\in Z$ *)*

(vii) $2\sin^2 x+\sin^2 2x=2$ *(Ans:* $x=n\pi\pm\dfrac{\pi}{2}$ *or* $x=m\pi\pm\dfrac{\pi}{4}$ *, where* $m,n\in Z$ *)*

(viii) $\cot\theta+\operatorname{cosec}\theta=\sqrt{3}$ *(Ans:* $\theta=2n\pi+\dfrac{\pi}{3}$ *,* $n\in Z$ *and* $\theta\neq(2n-1)\pi, n\in Z$ *as it makes* $\sin\theta=0$ *)*